Q) Solve the QPP by wolfe's method

max. $Z = 2x_1 + x_2 - x_1^2$

s.t.

$2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Solution:

Q.PP is already in standard form we will solve the QPP stepwise as follows

Step-I

Introducing slack variable q_1^2, q_2^2, r_1^2, r_2^2, the QPP can be written as

$$2x_1 + 3x_2 + q_1^2 = 6$$
$$2x_1 + x_2 + q_2^2 = 4$$
$$-x_1 + r_1^2 = 0$$
$$-x_2 + r_2^2 = 0$$

Scanned with CamScanner

Step II :

using Lagrangian multipliers λ and μ Construct the Lagrangian function

$$L(X, q, r, \lambda, \mu) = 2x_1 + x_2 - x_1^2 - \lambda_1(2x_1 + 3x_2 + q_1^2 - 6) - \lambda_2(2x_1 + x_2 + q_2^2 - 4) - \mu_1(-x_1 + r_1^2) - \mu_2(-x_2 + r_2^2) = 0 \quad (1)$$

where, $X = (x_1, x_2)$

$q = (q_1, q_2)$

$r = (r_1, r_2)$

$\lambda = (\lambda_1, \lambda_2)$

$\mu = (\mu_1, \mu_2)$

∴ Kuhn-Tucker necessary and sufficient condition for the existence of maxima of $f(x)$ are

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 2 - 2x_1 - 2\lambda_1 - 2\lambda_2 + \mu_1 = 0 \quad \text{--- (2)}$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 1 - 3\lambda_1 - \lambda_2 + \mu_2 = 0 \quad \text{--- (3)}$$

Defining $q_1^2 = s_1$ and $q_2^2 = s_2$

The complementary slackness conditions are

$$\left.\begin{array}{ll} \lambda_1 s_1 = 0 & \lambda_2 s_2 = 0 \\ \mu_1 x_1 = 0 & \mu_2 x_2 = 0 \end{array}\right\} \quad \text{--- (4)}$$

and

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow 2x_1 + x_2 + s_1 = 6 \quad \text{--- (5)}$$

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow 2x_1 + q_2 + s_2 = 4 \quad \text{--- (6)}$$

where, all the variables and parameters are constant.

Introducing non-negative artificial variables v_1 and v_2, we have wolfe's modified LPP as follows

$\min z_v = -\max z_v = -v_1 - v_2$

s.t.

$2x_1 + 2\lambda_1 + 2\lambda_2 - \mu_1 + v_1 = 2$ from (2) → (3)

$3\lambda_1 - \lambda_2 - \mu_2 + v_2 = 1$

$2x_1 + 3x_2 + s_1 = 6$ (7)

$2x_1 + x_2 + s_2 = 4$

$x_1, x_2 — s_1, s_2 \geq 0$. from (5) → (6)

we take $x_1 = x_2 = 0$, $\lambda_1 = \lambda_2 = 0$,
$\mu_1 = \mu_2 = 0$

we get

$v_1 = 2$, $v_2 = 1$, $s_1 = 6$, $s_2 = 4$

$$\therefore X_B = \begin{bmatrix} v_1 \\ v_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \\ 4 \end{bmatrix}$$

which is initial B.f.s. of the LPP (7)
To find final B.f.s. let us go through the simplex table

B	C_B	C_j / X_B	y_1	y_2	λ_1	λ_2	μ_1	μ_2	v_1	v_2	s_1	s_2	min $\frac{X_B}{\lambda_1}$
		C_j	0	0	0	0	0	0	-1	-1	0	0	
v_1	-1	2	2	0	2	2	-1	0	1	0	0	0	1
v_2	-1	1	0	0	$\boxed{3}$	1	0	-1	0	1	0	0	1/3 $\xrightarrow{\text{min}}$
s_1	0	6	2	3	0	0	0	0	0	0	1	0	–
s_2	0	4	2	1	0	0	0	0	0	0	0	1	–
$Z = -3$		Δ_j	2	0	5 ↑ max	3	-1	-1	0	0	0	0	min $\frac{X_B}{y_1}$
v_1	-1	4/3	$\boxed{2}$	0	0	4/3	-1	2/3	1	-2/3	0	0	2/3 $\xrightarrow{\text{min}}$
λ_1	0	1/3	0	0	1	1/3	0	-1/3	0	1/3	0	0	–
s_1	0	6	2	3	0	0	0	0	0	0	1	0	3
s_2	0	4	2	1	0	0	0	0	0	0	0	1	2
$Z = -4/3$		Δ_j	2 ↑ max	0	0	4/3	-1	2/3	0	-5/3	0	0	
y_1	0	2/3	1	0	0	2/3	-1/2	1/3	1/2	-1/3	0	0	
λ_1	0	1/3	0	0	1	1/3	0	-1/3	0	1/3	0	0	
s_1	0	14/3	0	3	0	-4/3	1	-2/3	-1	2/3	1	0	
s_2	0	8/3	0	1	0	-4/3	1	-2/3	-1	2/3	0	1	
$Z = 0$		Δ_j	0	0	0	0	0	0	-1	-1	0	0	

since all $\Delta_j \leq 0$, the solution is optimal and optimal value of the corresponding LPP (7)

min $Z_v = 0$

$$x_1 = \frac{2}{3}, \quad x_2 = 0$$

and the corresponding ~~&~~ solution of QPP is

$$\max. Z = 2x_1 + x_2 - x_1^2$$

$$= 2 \times \frac{2}{3} + 0 - \left(\frac{2}{3}\right)^2$$

$$= \frac{8}{9}$$